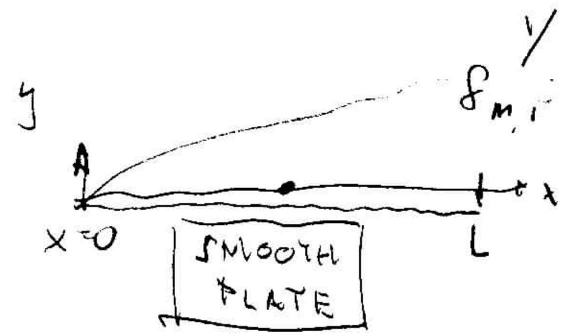
OK, so for a flat plate of length x we found friction coef. to be



Laminar

$$\left.\begin{aligned} \delta(x) &= \frac{4.91\,x}{\sqrt{Re(x)}} \\ C_f(x) &= \frac{0.664}{\sqrt{Re(x)}} \end{aligned}\right\} \text{ for } Re(x) < 5\times 10^5$$

Turbulent

$$\left.\begin{aligned} \delta(x) &= \frac{0.38\,x}{\left(Re(x)\right)^{1/5}} \\ C_f(x) &= \frac{0.059}{\left(Re(x)\right)^{1/5}} \end{aligned}\right\} \text{ for } 5\times 10^5 \le Re(x) \le 10^7$$

Recall : $Re(x) = \frac{x\,U_\infty}{\nu}$ Note $\lim\limits_{x \to 0} C_f(x) \to \infty$

We can define average values as

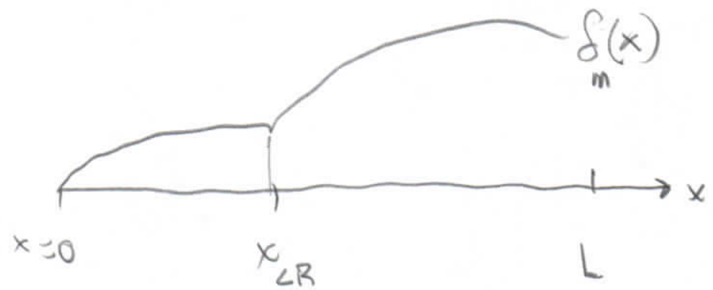
$$\bar{f} = \frac{1}{L}\int_{x=0}^{L} f(x)\,dx$$ in which case

All Laminar $\quad \bar{C}_f = \frac{1}{L}\int_{x=0}^{L} 0.664\left(\frac{U_\infty x}{\nu}\right)^{-1/2} dx = \ldots = \frac{1.328}{\sqrt{Re(L)}}$ for $Re(L) < 5\times 10^5$

All Turbulent $\quad \bar{C}_f = \ldots = \frac{0.074}{\left(Re(L)\right)^{1/5}}$ $\quad 5\cdot 10^5 \le Re(L) \le 10^7$

Lam, & Turb, conditions over the entire length of the plate.

But what about average values for leading laminar, then turb.?



Define

$$\bar{C}_f = \frac{1}{L}\left[\int_{x=0 \text{ laminar}}^{x_{CR}} C_f(x)\,dx + \int_{x=x_{CR} \text{ turb.}}^{L} C_f(x)\,dx\right] = \ldots$$

$$\bar{C}_f = \frac{0.074}{(Re(L))^{1/5}} - \frac{1742}{Re(L)} \qquad 5\cdot 10^5 \le Re(L) \le 10^7$$

See text for rough plates, or Hermann Schlichting (1979)

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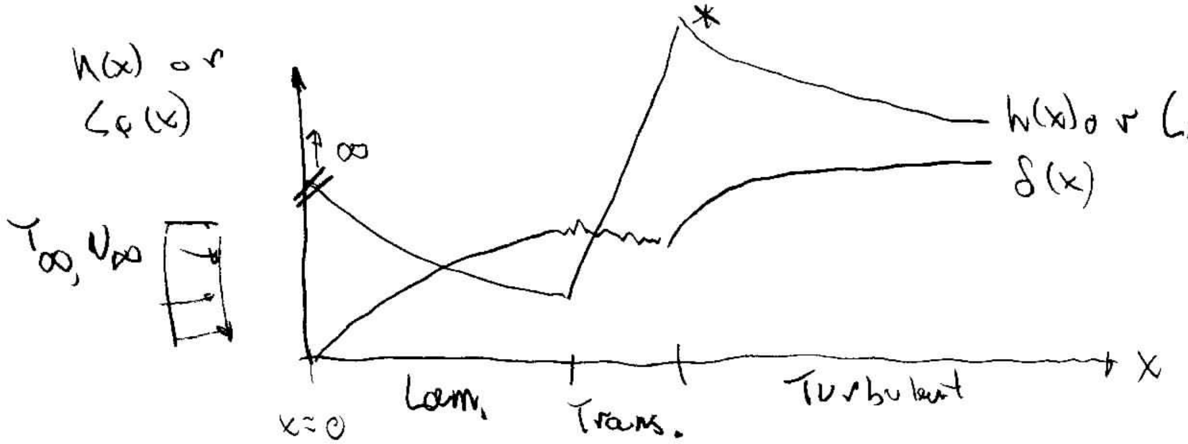
Now for heat transfer coef $h(x)$ and Nusselt # $Nu(x)$

local values! We found for a smooth plate

Lam:
$$Nu(x) = \frac{x\,h(x)}{k} = 0.332\,Re(x)^{1/2}\,Pr^{1/3} \qquad Pr > 0.6, \quad Re(x) < 5\cdot 10^5$$

Turb:
$$Nu(x) = \frac{x\,h(x)}{k} = 0.0296\,Re(x)^{0.8}\,Pr^{1/3} \qquad 0.6 \le Pr \le 60, \quad 5\cdot 10^5 \le Re(x) \le 10^7$$

note: $\lim_{x \to \infty} Nu(x) \to \infty$



* Largest value

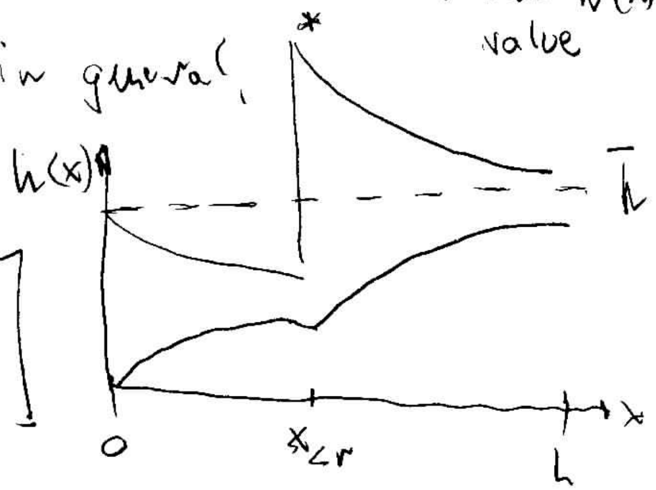
The laminar $\overline{Nu} = \frac{\bar{h} L}{k} = 0.664 \ Re(L)^{1/2} \ Pr^{1/3}$ $\quad Pr > 0.6$, $\ Re(L) < 5 \times 10^5$

Turbulent $\overline{Nu} = \frac{\bar{h} L}{k} = 0.037 \ Re(L)^{8/10} \ Pr^{1/3}$ $\quad 0.6 \leq Pr \leq 60$, $\ 5 \times 10^5 \leq Re(L) \leq 10^7$

Notice that

$$2(\ )(x) = \overline{(\ )} \quad \text{in general.}$$

* max h(x) value



For part lam. + part turb.

$$\bar{h} = \frac{1}{L}\left[\int_{x=0}^{x_{cR}} h(x)_{lam.}\, dx + \int_{x=x_{cR}}^{L} h(x)_{turb.}\, dx\right]$$

$= \ldots$

So

$$\overline{Nu} = \frac{\bar{h} L}{k} = \left[0.037 \ Re(L)^{0.8} - 871 \ Pr^{1/3}\right]$$

$0.6 \leq Pr \leq 60$

$5 \times 10^5 \leq Re(L) \leq 10^7$

For liquid metals $Pr \ll 1$ $\left(\begin{matrix} \text{mom.} \\ \text{diff.} \end{matrix} \ll \begin{matrix} \text{Ther.} \\ \text{diff.} \end{matrix} \right)$

so $\delta_m \ll \delta_T$

in which case you assume $\approx$ constant velocity inside δ_T of V_∞

Then $Nu(x) \simeq 0.565 \, (Re(x) \, Pr)^{1/2} = 0.565 \, Pe(x)^{1/2}$

$Pr \lesssim 0.05$

$Pe(x) \gtrsim 100$

$\boxed{Pe(x) = Re(x) \, Pr}$ Peclet #

//

one-size fits all (total curve fitting job!)

$$Nu(x) = \frac{h(x) \, x}{k} = \frac{0.3387 \, Pr^{1/3} \, Re(x)^{1/2}}{\left[1 + \left(0.0468/Pr \right)^{2/3} \right]^{1/4}} \qquad Re(x) \, Pr \geq 100$$

Good to within $\approx \pm 1\%$ of "correct" $Nu(x)$.

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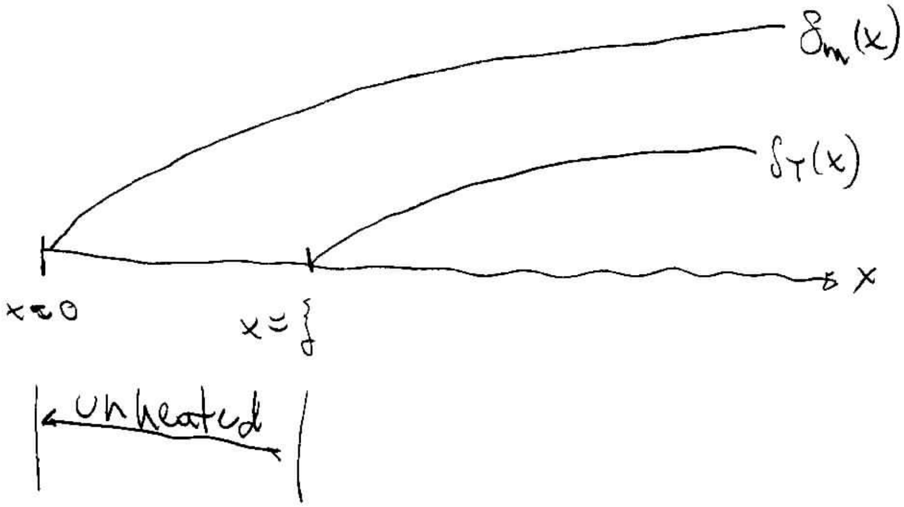
Unheated starting section



Laminar

$$Nu(x) = \frac{Nu(x)\big|_{\text{for } \xi=0}}{\left[1-\left(\xi/x\right)^{3/4}\right]^{1/3}} = \frac{0.332\ Re(x)^{1/2}\ Pr^{1/3}}{\left[1-\left(\xi/x\right)^{3/4}\right]^{1/3}}$$

Turb. if $x > \xi$

$$Nu(x) = \frac{Nu(x)_{\text{for } \xi=0}}{\left[1-\left(\xi/x\right)^{9/10}\right]^{1/9}} = \frac{0.0296\ Re(x)^{0.8}\ Pr^{1/3}}{\left[1-\left(\xi/x\right)^{9/10}\right]^{1/9}}$$

Laminar

$$\bar{h} = \frac{2\left[1-\left(\xi/x\right)^{3/4}\right]}{\left(1-\xi/L\right)} h(L)$$

Turb.

$$\bar{h} = \frac{5\left[1-\left(\xi/x\right)^{9/10}\right]}{4\left(1-\xi/L\right)} h(L)$$

//

But wait, there's more!

What about uniform heat flux (q''_{surf}) at the plate instead of uniform temperature?

Lam. $Nu(x) = 0.453\ Re(x)^{1/2}\ Pr^{1/3}$ $\quad Pr > 0.6$, $Re(x) < 5\cdot 10^5$

Turb. $Nu(x) = 0.0308\ Re(x)^{8/10}\ Pr^{1/3}$ $\quad 0.6 \le Pr \le 60$, $5\cdot 10^5 \le Re(x) \le 10^7$

then $q_{surf} = q''_{surf}\ A_{surf}$ ← heat transfer surface area

and $q''_{surf} = h(x)\left[T_s(x) - T_\infty\right] \Rightarrow T_s(x) = T_\infty + \dfrac{q''_{surf}}{h(x)}$

Examples to follow.